Rational Functions Practice Test - KEY

1) $\frac{\overset{3}{\cancel{9}}a^{\cancel{2}}b}{\cancel{4}\cancel{c}} \cdot \frac{-\overset{6}{24}b^2\cancel{c}}{\cancel{3}a} = \boxed{-18ab^3}$

2) $\frac{25-y^2}{9y^2} \div \frac{5+y}{3y} = \frac{(5-y)\cancel{(5+y)}}{\cancel{9}y^{\cancel{2}}} \cdot \frac{\cancel{3}y}{\cancel{5+y}} = \boxed{\frac{5-y}{3y}}$

3) $\frac{\frac{x-4}{x^2+6x+9}}{\frac{x^2-2x-8}{3+x}} = \frac{\cancel{x-4}}{(x+3)\cancel{(x+3)}} \cdot \frac{\cancel{3+x}}{\cancel{(x-4)}(x+2)} = \frac{1}{(x+3)(x+2)}$

4) $\overset{*}{18}a^{\overset{*}{3}}b\overset{*}{c^2}$, $24b^2c^2$, $4ab$ LCM $= 2\cdot2\cdot2\cdot3\cdot3$ $\boxed{72a^3b^2c^2}$

5) x^2-3x-4, $3x^2+3x$
$(x-4)(x+1)$, $3x(x+1)$ LCM $= \boxed{3x(x+1)(x-4)}$

6) $\frac{6\cdot2b}{5a^2b\cdot2a} - \frac{1\cdot a}{10ab^2\cdot a} = \frac{12b}{10a^2b^2} - \frac{a}{10a^2b^2} = \boxed{\frac{12b-a}{10a^2b^2}}$
LCM $= 10a^2b^2$

7) $\left(\frac{a}{a-2}\right)^{a+5}_{a+5} \frac{3}{(a+5)(a-2)} = \frac{a(a+5)}{(a+5)(a-2)} + \frac{3}{(a+5)(a-2)} = \boxed{\frac{a^2+5a+3}{(a+5)(a-2)}}$

8) $\left(\frac{5}{(t-4)(t+4)} + \frac{4}{t+4} = \frac{3}{t-4}\right)^{(t-4)(t+4)}$

$5 + 4(t-4) = 3(t+4)$

$5 + 4t - 16 = 3t + 12$ $\quad 4t - 11 = 3t + 12$ $\quad t = 23$

9) $\left(\frac{x-6}{2} = \frac{20}{x}\right)^{2x}$

$x(x-6) = 20(2)$

$x^2 - 6x = 40$

$x^2 - 6x - 40 = 0$

$(x-10)(x+4) = 0$

$x = 10, -4$

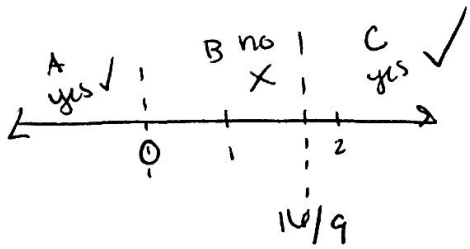
10) $\frac{2}{3x} + 3 > \frac{6}{x}$ $\quad 1.\ x \neq 0$

$\left(\frac{2}{3x} + 3 = \frac{6}{x}\right)^{3x}$

$2 + 9x = 18$

$9x = 16$

$x = \frac{16}{9} = 1\frac{7}{9}$



$x < 0$ and $x > 16/9$

A. $x = -1$: $\frac{2}{3(-1)} + 3 \overset{?}{>} \frac{6}{-1}$ $\quad -\frac{2}{3} + 3 > -6$ ✓

B. $x = 1$: $\frac{2}{3} + 3 \overset{?}{>} \frac{6}{1}$ $\quad 3\frac{2}{3} > 6$ no

C. $x = 2$: $\frac{2}{3(2)} + 3 \overset{?}{>} \frac{6}{2}$ $\quad 3\frac{1}{3} > 3$ yes

11)
$$\begin{array}{r} 2x - 4 \\ x^2+2 \overline{)\ 2x^3 - 4x^2 + 5x - 1} \\ \underline{-2x^3 \qquad -4x} \\ -4x^2 + x - 1 \\ \underline{+4x^2 \qquad +8} \\ x + 7 \end{array}$$

$2x(x^2+2)$

$2x^3 + 4x$

$-4(x^2+2)$

$-4x^2 - 8$

$2x - 4 + \frac{x+7}{x^2+2}$

12) Need same degree on top + bottom w/ leading coefficients simplifying to $\frac{3}{1}$ (Many correct answers)

Exs: $\frac{3x+2}{x+4}$, $\frac{6x^2+3x-1}{2x^2+5}$, $\frac{3x^2}{(x+5)(x-1)}$

13) Need the degree of the numerator to be one greater than the degree of the denominator (Many correct answers)

Exs: $\frac{3x^2+4x-1}{x+3}$, $\frac{x^3+2x^2-4x+5}{x^2+1}$

14) If a rational function is proper, the denominator has a higher degree, meaning it will grow at a faster rate. Therefore you will be dividing by a larger and larger number, bringing the quotient closer and closer to 0 as $x \to \infty$ or $-\infty$.

15) You find the values of x that are excluded in the domain because they would make the denominator = 0

16) $f(x) = \frac{10}{(x+1)(x-3)}$ Vert: $x=-1, 3$ Horiz: $y=0$

17) $f(x) = \frac{5x^2}{x^2-2x-15} = \frac{5x^2}{(x-5)(x+3)}$ Vert: $x=5, -3$ Horiz: $y=5$

18) $\frac{(x+3)^2(x^2-9)}{(x+3)^2(x^2-11x+24)} = \frac{(x-3)(x+3)}{(x-3)(x-8)} = \frac{x+3}{x-8}$

19) $\frac{x+7}{(x+7)(x+3)}$ x cannot equal -7 or -3

20) $x-2 \overline{)\, x^3-4x^2+2}$

x^2-2x-4

$-x^3+2x^2$

$-2x^2+2$

$+2x^2-4x$

$-4x+2$

$+4x-8$

-6

$x^2(x-2)$

x^3-2x^2

$-2x(x-2)$

$-2x^2+4x$

$-4(x-2)$

$-4x+8$

$x^2-2x-4-\frac{+6}{x-2}$

21) $x-2 \overline{)\, 3x+4}$

3

$-3x+6$

10

$3(x-2)$

$3x-6$

$3+\frac{10}{x-2}$

22) $ax+3 \overline{)\, 12x^2+30x-12}$

$6x$

$ax(6x)=12x^2$ therefore $a=2$

23) $\frac{3x}{(x-6)(x+2)}$

Verti: $x=6, -2$ Horiz: $y=0$